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APL NEWS

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APL WORKSHOP

An APL workshop r,raa hel-d May 16-18 at Queenrs University. Kingston, Ontario, under the sponsorship of the Departnent of Computing and fnformation Science and the SIGPLAN Tech- nical Conmittee on APL (STAPL). The 35 invited participants included representatives from some of the manufacturers who

support APL (IB!,1, Burroughs, Digital Equipment Corporation), several time-sharing services and tt\e academi.c community. The major topics discussetl were the development of an ApL standaral, a standard information format for workspace i[ter- change, and the need for language extensions in the areas of general arrays, control structures, and error handling and event controf. A fuller report of the workahop activities

and a list of addresses of the participants wilL be submitted to APL Quote Quad, or is available on request from the rrork-shop organizer, M.A. Jenkins, Conputing antl Information Science, Queenrs University, Kingston, Canada. K7L 3N6.

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CUBES ,,4rn6eaat Gfilngn

(r) t(oc)=N, ,N

The st'nbol t will be used to assert that the A?1,

expressibn imed.iately following it is a well-defined Iogical array with no zero entry r so that tE,fPf is

equivalent to t^/,ENPR. We shall use zero origin throughout.

we shall be concernedl with arrays d satisfying the following three non-indepenalent conditl.ons :
tC€ rlVt 3 t(rlV*3)<e
so that d is an affangement of the elements of 1ly*3 in the form of a cube .

Our deflnition of a magic cube is far more stringent than that usually given. For us a rnagic cube of oraler tV is
an array C satisfying (I) and such that, in each of the thirteen possible directions, every set of 1y elements of C forning a straight line in the cube c has the s€une sum. In the usual definition. even when the cube is deacribeal as "pand.iagonal", o41y seven directions are considered. namely

the three orthogonal directions and the four alirections of the space diagonals. Occasionally our magic cube has been

trlterreal to as a perfect rnagic cube. U A pa[tliagonal cube is an array c such that for any

choice of i i-egets XtIZ the arlay x0t0ll0t1lz0t2lC is a magic cube.
An associated cube is a rnagic cube C for r hich
(2) t^/(-7+x/pc)=(,c)+O,c

The theory below provides a general method for the construction of magic cubes, including the following four

examples :

(3) (3e7)p7L7l(e g+s spq s 6)+.x(3p?)117*3 (4) (3ps)p8ro 1-2s7 6 5 4[8](s s+e spz 3 4) + . x (3 p 8) r r 8 * 3]

(5) (3p9)p9rl 2 o 3 4s I6 7[el(e e+e spz 3 5) + . x (3 p 9) r r s * 3]
(6) (3p11)p111111(3 313 5p3 4 5)+.x(3p11)rr11*3

of these examples, (3) is assocl-ated. (4) is pandiagonal, and each of (5), (6) is an associateal

pandj.agonal perfect magic cube.

r-

THE THEORY a
ft is convenient to encoale the efements of C antt of
1iV*3 by using the matrices Mf a\d MC defined below:
(7\ ntr + (3p)rr,['*s EMC=(3p,V)r,C

Observe that (8) t(11r*3)=lt'.rtvr tc--(3p ') pN lr,rc
and that (9) tMI<1N tMCelv t((pMC)=pur)t (pur)=3,Nt g Taking ff ,aa 3=oX). to be a suitably chosen matrix, subjeet to the constraints below. we define C by using (8)

and (7) in conjunction with
(I0) Mc + N lK+.xMI
so that the array C is in fact defined by
(11)

modulo
(L2',t

{- (3plV)p/trfl l?lr. x (SoIy)rrlvr.3
Then C wlll satisfy (1) if and only if 1(is invertible
IV, which is equivatent to the condition

where DE? is the divisor function.

El=N GCD DEI K

determinant and cCD is the greatest corunon

cube

(13) t^ /,a=N GCt l(n. x1-(3p3)rr13
wherein columns O 2 6 8 correapond to the four space diagonals .
The cond.ition (2) for C to be an associated cube (if
magic) will be satisfieal if and only if
(14) t'^/7=Nl+/K

The array e satisfying (1) will if f satisfies the condition

be a pandiargonal ma\,

independently on the

sets convenient to take](to be
(rs)

Since (13), (14) depentl only and of numbers in each row of .K, it is

a circulant matrix:

(which nay also suitably chosen

N 1- 0 2 103 3pt/
be written ff<-3 3+3 spl/) \rhere I/ (t3=py) is a
vector.

lf t^/0'2 3 sllv and if we define

we find t1=nl+/V and t1=lylsx1rF so that -tr has the mod It

inverse (3 3+3 5po 2 7x!+R) anal consequently]l satisfies (12) and (14) and c satisfies (1) and (2). Since, except for coLunns 2 6 8, (13) is easity verified, it fotlows that in this case (t0*2 3 slll/) (15)

yields an associated magic cube, \,rhich is pantliagonal if also t0t7 liy. By taking iV<-7 or.v.-11 in (11) and (16) we obtain the examples (3) and (6).

If w/o=2 3 s T ll'/ it is tunpossible to satlsfy (13)

.

but I'e may obtain examples such as (4) and (5) by incorporating the step MC<-W\MC\ where tr is a rearrangement of rlv such that, for any integers A anA Dl

(17) to iIt l, irnpf ies L(+/v)=+/WtN I A+Dx 1N 1

and, in the case t0=311V, by rnodifying 7.

we close with seven problems, of which the last two
are by far the most challenging.

CHALLENGES TO THE READER

A. construct a magic cube of order 25.
B. prove that +.2o4\=+/,(x6u) /lo)(vbu) /111(ziu) /121c,
where c is given by (4) and xtytz'J are integers such that t2=r/u+(i.a)€o,J. Interpret this result, and fintl other Eirnilar results.
. c. construct a four-dimensional analogue of a magic cube. $'b".*" that the linear sums are the same in no fewer than forty different directions,
D. Construct a strongly associaled magic cube of oraler 8'

where a magic cube c is strongly associated if (18) t^/ ((\x/pc)=,c) v ((rx/pc)=0,d)

E. construct a 5x5x5 cube on 1125 in which more than 100 lines of length 5 have the same sutn.
F. Determine whether or not there exists a magic cube of order 15
G. Determine whether or not there exists a
four-dimensional magic'cube whoae order is less than 16.

aBETI?(?!86)ppdBETT

what capital city was tranaformed to Eastern capital
by -20 7

'$p'ur ta <- 3 343 5p r <- (13)r,R * (31-t/)xLrV+3

APIJ progranner Bob Hankey of DSG, 133 South 36 Street, Philadelphia, now combines progranurdng with the perm-anent repair of typebaLls, a business he invented when his only APL typeball suffered a broken biscupid.

ON FUNCTION DEFINITION

SQRI : u* .5

A forn of function definition particularly suited to exposition is defined. and illustrated below in &cerpts frorn Iversonrs EteJngllt-ary Anetysis (ApL press, 1.976). We rroulal be pleased to publish any intelesting and efficient ApI program gapable of fixing the definition of a function presenled to it (as a character vectort in the oar form here iescritea. --

-

Consider a dyad.ic function _F defined. informally as foll.ows: the result is the sun of the right argument and the product of the left argument and the square of-the right argrunent. fn other !rord.s, the result of the expression.f F y is I+X'1*2i specifically, 2 F 3 equals g+2x3r,2 or 27.
A formal definition is one which can be interpreted by a _mecha GEI- application of known rutes. r"q"l;il;- ;'" judgenent or subtle interpretation. f'or exarnple, - the function F defined informally above can be defined- formally asi

F'.a+o.tut 2

usinq the following rules: to interpret any expression of the form .tr F y, substitute the first argune;t x for each

occurrence of c, (the first letter of the Greek alphabet) in the expression. and tEEifEst argument y for each occurrence of a, (the last letter of the Greek alphabet). For example, the steps ifi"-Ehe interpretation of th6 expression 4 F 3 can be shown as follows:

4F3
3+4x3*2
39

If the sl4nbols d anal &r do not both occur in a definition, the function defined is monad.ic. For exampl-e: PII fME S | 3 .74!6xo)

The colon in a function definition may be read. aloud a9 "is". Thus, I:a+.J rBay be read as ',F is o plus &j".
A variable which- ls asslgneal a value lrithin a firttction definition is local to the function.

.

A function definition is said to be recursive if the function being defined recurs in the expresSlf6i--?le-fining it. This notion nay be faniliar frorn inforrnal definitions. For example, the power function .tr*rv may be said. to equal .f times.f*ll'-1, and the factorial function lilr may be sjid. to equal

ryx !lY-1.

-.

Let us attempt to define the factorial function -plc in ' rts manner: v

EAC :uxPAC a-7

To- interpret the expressiot FAC 4 we would. proceed by substitution as usual:
EAC 4
4XEAC 3
4x3xFAC 2
4x3Y2'EAC 7

It is clear that this procedure can be terminated meaningfully only if lre know the value of EAC X for some value of the arg,ument X. In this case, FAC 1 is equat to 1, and with this knowlealge we can terminate the inteipretatl-on

as follows:
4x3r2x1
24

In general, it is necessary to know a gecond. expression for the function (in thie case the simple expression 1) and the condition under which it is to be applied. (in this crase when o=1). The recursive definition of factoriat therefore requires the following three pieces of infornation:

utF a-!

7

The primary expreasion:
A proposition 3
A secondary expression t
In a formal definition these three tlata are presented foregoing order r,rith colons separating them. Thus: FAC:uxEAC a-!,a=!,7
FAC 4
24

Area of a cloaed !!ggre. A fi$rre is ctosed. if the last!6i-nt -Tn- its-matTix-- represeitation equals the first.

However, if a figure is known to be ctoseal, Lt can al-so be represented rnore brielly by dropping the]-ast point and requiring that the connection of the points be cycllc. i.e.,

tbe last connects to the flrst. Flgnrre 5.19 shows the pentagon so represented bl^ the following s-column matrix 3

P

7 70 6 3 4
25863

The area of a closed polygon so represented is given by the foUor,ring Eingle function :
AREA t.5x+/ (xl1 10.0)-xll 00.,r t5.9.61

perimeter of a closed figure. If p is a two by ll/ matrix ElEEEnEin!-th-e-Z-oordiiates of the /y vertices of a ctos r. polygon, then the displacements from vertex to vertex r _
given by the expression D-p--!6p. For example, if p is t)G matrix of Figure 5.19, then IV is 5, and:
P

7 70 6 3 4
25863

- 1OP D
4 7 10 6 3 3 3 -4 -3 7
3 2 5 8 6 -1 3 3 -2 -s

.- .- Moreover, the length of any displacement (according to the theorem of Pythagoras) is the square root of the sum of !h. squares of the displacements along the .tr and y axes. consequently, the lengths are given bt EeE! +lD*2, lvhere

EQEIte*.5. .The lengths of the sides of a polygon are therefore given by the fotlowing function appiied-to the natrix representation of its vertices:
LP tEQE! + I (tt -- lQu) r,2 t5.9 ,7 1

The entire set of functions developeal for handling polynomials is collected below:
Pt+/axut 7Ns,
N:+/a=u
PLUS | (ML 4 e) + (ML<- (N a)f N tt) t t)
MII,IUS I o, PLUS -.t

ffMESt(axt+a) PLUS o,a fIMES L+a;o=ll!ntO
,UliTOi (1{&,r1+cr) .o. INI) 7+d MINAS ax1+0r+1+oi (iVo)>IVrrr: r0
LINrO tQ(6e.) INIO 6@
CU-PT:(-+/L\0=0o)1ar
PVRAr(ct P 7lu),a PVRA \+uto=N@.rO

Many of the properties of a function can be exDrer I

in terms of tbe properties of the component functioirs uE6dl in its definition. Thus if E:(Fa)+(ca), then (as shown in

sec 2.3 and. 3.3) the ad.d.itive function for fl is given by

tH:(o.tFa)+(agGu) 1 and the d.erivative is eH:(p?;)+(Acu). Rules.for obtaining the derivatives of composite funcaions are l-isted below, using Af and 2C to denote the d.erivatives of 1 and C, and lE and. trd to d.enote their inverses:
NAME COMPOSITE FORM DERIVATIVE

SUM (Eu)+Gu
PRODUCT (Eu) xGa
RECIPROCAL +Fu
COMPOSITION F C u
INVERSE IF a
POWER a* Il

4.6L If 7 is any two-element vector, sho\d !hat:

a) 2 o +/v
-- /x /7 2..o v

(PF u)+2c r.r [6.1.1]

((Eu)xPG @)+(DE @)xcu t5.L.2l -(28 u.)+(Fu)*2 t6.1.31 (2F Cu)xDC ar t5.1.41 rDF IF a t 6.1.5l

Nxt *lt-t t 6. I.6l

b) r o +/v
+/xlj L67 2o.o V