APL
PRESS
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lhie eeeond iesue of APL Neue p?esent6 a T.anguage ertension pnopoeal by Doug Fonkee, and a
neu column by Jeff shaLlit, to be deooted to reereatione, Corneapondence on anv of these featutes ui.LL be uelconed.
Beginneys in APL criticized the fitet issue beeauee it contained nothi.ng of intereet to them. Ihe eece?pt on Read'Lng APL ie offez'ed fon e,uch leaders! bomment on it uould be appreeiated' Many readere aeeuned that teeei.pt of the
firet iaaue i.npli,ed that they uould conti.nue to
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EDUCATION CONFERENCE

A conference on APL in Engineering Etlucation
was held 1dat tlreh Sagamore Sagamore Conference Conference Center' August
78-20. 0rganizetl by H.ofessors Wilbur T.,ePage ancl

Garth Foster, and sponsoreal by the Department of Electricaf antl Computer Engineering of Syracuse University, the conference drew about fifty partici-
pants r mostty from mathenatics and engineering faculty in Canada and the northeastern United States. Further information is avail-able from the organi-zers; in particular, they are preparing a report for submission

to APT, Quote-Quad.

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Definition. Tr^to arrays 4 and. B satisfy scalar conform- SiEET-IE lheir shapes agree or if at- least one is a scalar

or one-element vector. I propose extending the condition of scalar conformabi-lity to a thiril case where the ranks agree and ^l((p/)=pB)v(I=pA)v7=pB. The shape of the result in this case is (x(p.4)LpB)x(p1)[pB.
It j-s alifficult to express the result in conventional

APL (thus iltustrating at least one use for the extension, albeit circular). Stated informally, the dimensions which are equal to t have scalar extension applied to them. For example, if A<'7 3 7pr2! and B<-7 1 5p135, then A+8 <--+

4[;;5p1]+B[;3p1;].

The above expression is in 1-origin, as are all expressions in this article. It is worth noting that rnany of the uses of this extended conforrnability replace

origj-n-clependent expressions with origin-independent ones. A ner,t function. We now def ine a furiction o such t:har- AuB <--> T(:Ttl U'$E);E: Thar is, (.) expands the shape vector of B, !-'pcting 1 rdhere there j-s a 0 in /. For example, if

+t 7 203, then 1 1 0 ! OuB {--} O 1 7 2 LpB. Notice that no

change is made to the nurTrber or order of the elements of Bt the shape is merely expanaled.

This new function is useful- in proclucing arrays satisfying the extended conformability rule. The usefulness of at in this article suggests that it woulct be worth
implementing as a prfunitive if this extended scalar conformability were adoptecl.
Examples. If MX<-3 3pr9 and FC<-13, then:
Mx+(pMX)pVC 1--> Mx+o 1.@Vc (adding matrix to row vector) MX+Q(opMx)pyC <--, MXt1, ouVC (adding matrix to colunn vector) MX..+VC <-+ (1 1 ouMt)+o 0 luVC (outer product)
MX+.xVC a"> +/MXxo TUVC (inner product)
Of course these last two examples do not represent extendecl capabilities, but rather show the close

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relationship of this definition to other APL concepts. '' Hottever, they point the way to soLutions to problems rel\-7id

to inner and outer proaluct which are currently awkward. For example, compute ,4+.crB, where q is some scalar function but

not a prinitive function. For example, if AdB <--, IfAxB,

then:
MX+.o.VC <--' +/71 7 2 2\MXo.+VC
+ + / 7l MXxj LuVC

Consistency with other APL concepts. This definition of conformability is used in some implementations of inner proaluct LPakin, 1972J. Two arrays are conformable if their inner dimensions are equal, or if one or the other equals 1. In the latter case the array is extended in exactly the manner proposed here. Notice that this operation also benefits from the ne\r function (,r. For example, suppose that P is a vector of primes and that ly' is a vector of integers. Then (xPL,l(7,pM)pM)/M gives the elements of M that are relatively prime to every element of P. The phrase (I,pM)pM

can be reptraced. by the slightly neater phrase o !uM, so that we have (xPL,lo r@M)/M.
In those cases \dhere the extenaled conformability

coincides with present conformability, the resuLts are iiienticaL with the conventional- definition. This wou1d. seen

to be alesitable for any such extension, although a similar proposaL lBreecl, 197I] diat not have this property. \.,

Implementation considerations. The function a.r is easily expie s 5eA-fn presE;E-E[=n?I-is easily implementeal.

Implementations r,rhich allow multiple references to the same data structure and separate references to data vector and

shape vector would be especially suited to the implementation of 0r. The extenaled. scalar conformability \troulcl pose few difficulties, being essentially a matter of nested loops rather like those used j.n inclexing; indeed it
vtas no coincid.ence that I usecl indexing in my exarnple definition.

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REFER,ENCES

Pakin, Sanalra, APL 360 Reference Manual, Science Research Associates, tgt77pi46 Trl-- L.M. Breeal, "ceneralized Scalar Extension", APL Quote-Quad, VoI 2. Nunicer 6, March,.197I, paqe 5.

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This is the first APL-PLAY column, ancl I would like describe some of the kinds of things r.ve rrill be cloing 3 A. Problems -- consisting of both writing functions to perform specified tasG)-Eitf

explicate a given function or expression.)
hrhat is the significance of the expression
matrix M? For any atray M?

(tlef ining
E391ng (to

For example,
^/M=L\M for a

B. Games -- listings of games you can play on an APL machine, with suggesteal moalifications left as problems. C. Proofs -- of new or oLcl identities using APL notation. For example, I coulcl use a neat proof that /QEQ6:{--}AtBlQc. D. APL Trivia -- including notes about APL anal its current impl-ementations, known bugs (such as a domain error for

=/tABct), and novel uses of scan, inner and outer products, anal other functions.
Except for this first colunn, which is a potpourri,

each coLumn will have sorne unifying theme such as number theory, graphics, or geometry. I invite your participation.
f(r, eirr" a clear explanation
fol lovtinqt embodirnent of the

of lines 2, 11, and 12 of the popular game of Masterminal:

SIAR!TNG OVER.I

',(qIY),t TRIES.t V

V MAS ?ERMIII D .lI0 .IABLE zCUESS zfNPUl zN t S
t 1l DI1-I
I 2J SIARI : I ABLE<-Q1r (StO?,9p 1 + o C)LORS) 1- !+ t (7 t p C)L)RS) t S L)T S t 3l rV* 0
t4l GUEss tN<-N+7
t5] GUESS<.,TABLEIS<.?7+PTABLE;f
t6] TABLE<-(SI1,I+PTABLE)ITABLE

l7l l<-,(c0L)RS,t |)IGUESS;f t8] IEIT?ER NUMBER AF BLACK FOLLOTIED BY NUMBER OF VHTIE,I

t 9] INPU?<-N
t10l -,(sL0TS=INPUtlIf) /END
t 111 rABLE<_(INpUtLrT=IABLE+. =cUESS) ITABLE
lr2) IAB LE<- ((+ / INPUT) = (+ / \27TABLE. . = t r + o C)L)RS) + . L +lcaEs S ..=17+pC0L)RS)IIABLE

t 13l -+ (or!+ IIABLE) / AAEEL t14] IYOU EAVE MADE A MISTAKE. WE ARE

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2. The golden ratio, G, is the positive root of the quadratic equation 0=X-r1 1 1.
a) Write an APL expression for G using any ApL prinitives,

decimal points, the digit 5 (any number of tirnes), and no other tligits.
b) Repeat part a for the negative root, G.
3. There are two conmonly used ways to represent lists of names of arbitrary lengths: as the rows of a matrix, or as segments of a vector separated, by one o! more spaces.
The canonical form for a matrix of names has each row left j us t:lfiEl-Eitl l-i-ini:num wiclth. The canonica]. form for a vector has a minimum nurnber of spaces.
a) Write a function CANON, to return its argument in

canonical form, whether it be a matrix or a vector. \-/ b) Write a function C?NVERY, to convert a matrix in

canonical form to its vector equivalent or vice versa, depending on the rank of the argument.
q. Write the briefest possible " seLf -reproalucing function!' S8F, whose execution produces a print-out iclentical to that produceil by VSAFIEJV,. t/there the brevity of a function E is

measured by the expression +/,v\r 'rolcR rFr. Except for its length, the follo$ring example servesi v z<-sRF t1l Z<-(s 3 er [1][2]t3lr),(s 3p' ')'(DCF t,9.R.t'r) ' [1] I I

121 Z[1;5J-crVr
t3l Z[5;5]<-rVl
V

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I(EADING /aanc APt E.lemenlaegr .&tn/ya
Learning to read expressions. The only effective \tay to Iearn a language or notatlon is to use it. Use of the notation in the exercises thus far shoulcl provicle familiarity with each of the functions and the slzmbols usecl to tlenote them. The reader hdy, however, stil1 have difficulty in interpreting compounal expressions involving a nundcer of functions.

The main point to be appreciated is that the fert rules
and clefinitions given in the preceding five pages are complete and mechanical in application, no subtle fiEEifire-tation iE-'iie-ftiEil- rn this respect, readers familiar with conventional notation nay find their greatest bar to learning psychological; one is so accustomedl to the need for subtle inlerpretation that it is ctifficult to apply precise rules mechanical"Ly. !'or exanple ' one is used to the fact that X(x-3) means something quite different from f(x-3), and that values of variables cannot always be substituted for the variables in an expression (thus if x is 3 anal y is 4, the expression x.Y is equivalent to 3I but.not to x4 nor 34,
v ',ch norfially denote something entirely clifferent). wi
Mechanical intetpretation of an
carried out in the following steps:

2. Execute the first function
argument, or arguments, in
value obtained.

3. Repeat steps 1 andl 2 until
in the expression.

expression can be

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1. Determine the first function to be executed' According to itern 9 at the beginning of this

section, this function is the rightmost which can (subject to the rules for parentheses) be executed. and replace it and its the expression by the
no nore functions remain

Consider, for example, the evaluation of the expression itr = + / (1 + 1 /y) x 0 = (1 + t /V) i f for the case where ,V<-6. The steps in the interpretation are sho\'tn below, with a line under each portion being evaluated:

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iV=+/ (1+ rty)x0= (1{r/V) I tV
lV=+/ (1+rlt/)x0=(1+r4) I 6
it = + / (! + | N) xo=(1lg_1_2_9_t_g) I 6 N = + / (r ! r N) x 0=1_A_C_l_g1g
r/= +/ (1+ 1lV) x 0:g_g_q_2_1 I'l=+/(7+tN)x7 110 o N=+/lL!9--!-2-g-!-9.)'1 1 1 o o
N=+/7 2 4 5x1 1 1 O 0
d = tl.1-?-E-q-9.

4=o
.q=0

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. With experience one of course collapses the process by combining sirnple steps and by executing iirdependeni expressions in parentheses simultaneously. foi exampie:
N=+/ (!+rN)x0 =

(1+ 1IU) liV

6=+/r 2 3 4 sx0:1_2_3_!._510

i Finally, one learns to grasp the significance *t entire phrases. For example, in tie unclerliiecl ptriise ;i

1 the last expression above, it is clear that 1-2 3 4 516 i lields remainders on dividing 6, anct that the entire] unclerlinecl phrase therefore yields 1,s and 0rs, the 1's

I indicating the elements of the vector ! 2 3 \ 5 that are

I divisors 91 6. Moreover, the vector ! 2 g q 5 clearly includes all possible . d.ivisors of 6 (except o itseff). Thi succeeding rnultiplication by I2 g 4 s therefore yielcls a
I vector which incl-ucles only ieros and the divisols of 6. I Hence the summation (+/) yields "the sum of al1 clivisors of 6. This sum is finalty iornparecl with 6 yielding a 1 if a

equals the sum of its (-other) - divisors.
More generally, for any positive integer value of ty it appears that the expression yielals 1 if lV is a perfect

number, that is, a number which equals the sum of it;:oTEA divisors. In other words, the expression is a test for perfect numbers. Moreover, the actual evaluation can now be performed by an efficient shortcut; since the expression to

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the right of the equal sign yieltts the sum of the clivisors, they may sinply be listed anal sulnmed. Thus, for iV<-27, the
i:]"nt"="t""

becomes 2'7=+/7 3 e, and clearly yielals 0.