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APL NEWS

APL I

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ouz, nai|. eontinuee to bring i.n requeets foz' infoz'nation
on the use of APL in a ui.de oa:rietg of a:reae. Recent eramplee ineiude eonmevciaL Lpplications' population onr"ti.". crLttcal path anallllis'' compute! seienee and "mathenq.tics at the high schobl l'eoel, APL inplenentqtions on miero-computez's, and cioiI. engineez'i'ng.
Mana colleapondents ask to be put,in touch -uith othe:rs uorkin'g in the sane area. lle uould be pleased to paee on ot' pub-Lieh statemenltl af uoz'k i'n progrese.
Ihe APL 76 Confexence Ptoeeedi.ngs are nolt auaiLable
fvon ACM, Bo$ L27'05' Church St:reet Station, eu yo:rk
iozqs. 'nlthaugh conmerctal data prace.asi'ng ^fotns the nai'n

inZil, ti| i"iLeedi.nge touch on a- uarietv of othet' top.ie s'

+"Lillat"s thi irga"i\ation of APL Libraries, pz'ogranming te"hiiqu'es, and -teut editing, as ueLL qe eertain Language

questi,ons.

g"// g^aait

Parallel to the self:reproducing function dit.u,,"kn
NL2 is the self-reoroducins expresslon (sRE),- defined iniorr" i rv tv p-hil nurat* is nan express ion wh lch when tvped in from a'teiminal outputs what was typed inrr' Use of the eiu.ui" function perrnits the following formal definition: A character vector d ls. an SRE if;

(L) ^/C=aC, and (2) t'/-cetABC. ,,zLLB-7 . ..ZLt .
For example, if FAKE<'tEAxE t,'then

^/FAKE=7FAKE

results

in a 1. but FAxE ls not an SRE because of condl tion 2' Three Lxamp tes of sRErs follow, the last of which ls the
sho rtes t known:

r''t1 1,26p1131r[1 1,26P113]
rO26pr4p1,3p r I r 1S26p14P13P I

ro22p77e t t t lq2zprTo j t I
@r* lr:ss rrzz

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The following questions present themselves: ls there

a longest SRE? Are there only a finite number of SRE|s? We will show that the answer to both these questions is "norr.
.. ln any particular implementation of APL there are only

L.'inite number of SREts because there ls on.l y a finlte l 6unt of space available. ln the APL languaEe, however,

where we are not concerned with such limitations as !t& and

!PP, we can find an lnfinite number of SRErs, The fol lowing function produces a different SRE (of length 50+gxI/) for

each positive integer argument 1V:

v z<.AcE lt t1l Z<-10(50+Bxlv)p(25+rrxll/)p" I10(42+8xp',("1,,Up1),,)p(Zt+

4xp' ,

(r1,.'lrp1),r)p I ' ' V

The method of construction used in A?EN is best seen ln examp les (with [Ptl+60):
ACEIT 7

10(42+8 xp 1 1)p(21+4xp1 1)p I I r lS(4218 xp 1 1)p(21+4x01 1)pttt

'AGEfi 1
1O(42+Bxp1 1)p(21+4xp1 1)p''r10(42+8xp1 1)p(21+4xp1 1)p t
AGEII 2

1O(42+8xp1 r 7)p(2!+4xe7 !
1)p ' ' ,
AGEN 3

1O(42+8xp1 7 7 7)a(27r4xp7 xpl 1 1 1)prrr

1)p ' I ' 1O(42+B xpt 7 7)o(27+'+xp7 1

1 1 1) p ' ' | 1o (4 2 r 8 x p 1 1I7)p(27+4

8050

o

p X<'AGE 1,0 O0

^/X=lX
Problems:

(1) Can you find an sRE d such that 22>pC ?
(2) Can you find a character vector c that is not

immediately self-reproducing, but which generates itself after two or more appl'ications of the execute function? For exampfe, is there a c such that ^/c=erc but that -^/c=tc ?
(3) ls there an sRE of odd length?
Answers to orob lems oosed in NL2.
Prob lem L (Mastermlnd). Line 2 defines a matrix LABLE of dimension 1296 4 conslsting of all possible four-digit arrangements of the digl ts !.2.3,4.5.6. This table represents all possible cholces of the six colored pegs in

four slots. This is a typical example of the use of the encode functlon in dealing with arrangernents.

The user enters in line 9 the number of black pins (to \' indicate the number of pins correct in both color and position), fol lowed by the number of white pins (to indicate the number of pegs correct ln color, but incorrect ln positlon), Thus in line 11 the inner product IABLE+,=GUESS determines how. many pegs are correct ln both color and positlon for each entry in IABLE.
ln line l.2 we consider the quantlty +/IIIPU?. This number corresponds to the number of pegs correct in color, without regard to positlon, The expressions +/l2JIABLEo.=17+pC0L?RS and +lGUEsS".=\TtpCOLORS form rrfrequency tab,lesrr indicatlng how many pegs of each color occur ln any arrangement. The +,1 inner product then rrmatchesrr the number of colors
ident i ca I in both arrangements.
Problem 2 (Golden Mean). c<-,5+.5x5*.5 and e<--:.5+,5x5'r.5 or G+.5-.5x5*,5.

Problem f (Representations of name lists).

v Z<-CAN0N XiA

t1l .>(!=ppx) /vECT0R L2l A*(-I+(' ' =0x)rl) O,r

t3l z<'(^\vl*t t)/A

[4] 'o t 5l VECT0R:A<.t t rX<-t ' ,X t6l z<.1-+(Av76A) /X e
i Z<-CO VERT X,I iD ilfo
t 1l aro<-7 \t
l2) -+(1=p px) / VECT)R
t3l z<--1"+(,(7++/x.t ') ". >r 1+-1 +px)/,X,' '

t4l -)o

t 5l VECI)RtI<.(x=t t') / rpx

i 6I D<-(I ,7+p x) -!ro ,I l7l z< ((pD),1/P)p(,D" '>rl /D)\,(x.t t)/x v
Problem 4 (Self-reproduc.ing functions). G,L. Stoddard of lnterstate Electronics Corp. points-.out that the shortest self-reproducing function has the canonlcal representation 2lptBAr, where the global variable.4 is pre-speclfied as A<-3 7pt v B[1] A vt, However, if we disallow the use of global variables, then the shortest known function
is:

I
I

I
I

VS

t1l (3 5 p'
v

vtll ,), 3 -36+Ocn's,

AN dO) fral ,llaflaati
COMPILER Gontaol AilaG"afi.

\'
The following functions provide a solution' to the problem posed in the first newsletter, fixing the definition of any function presented in the ao form defined in lverson,
E lementarv Analvsis, Name conflicts wi ll not occur if the user avoids names ending in 9, and any name assigned a value within the function defined wi ll be localized, The function DEI is used as follows:

DEE DEF PLUS.o.+@ Bt(Z,O)+0,La-B u-Itu--ot7

3PLUS 4
'7 7 5 10 '510 5 1
V,EFt!] V
v DEF iB9:,89 ilIO;ALPH9;X9
t 1 I Eroa-7

12) -0(1 o ^,=|qa|ex9<,L)/'rg[(xs=r'orr)/rpp19]+'r{!rtr t3l ->((v/^/t t=xs)v-v/ 2 4 e7+pxg<-t | | MATS t t ,89<',x9)/ilg

t 4 I ALPHs<- t AB cDEF CHr JKLMN oPQRSruvtrxv Z4EqDEECE!{.ZLAUA?OECTU yaz!207234s6'7 8s I

tsl Eg+tzg<'t ,((tateBs)/ ' -rs '),.rst1;l,(tuteB9)/t 19 t t6l se+89,,((o 1+Be)v.tt t)lB9<-(0 o ptl) SPECIEISOBS
171 '>(2=7+pXs) /Ls

t8l E9189 RJNs I Zsa-' ,Xgl4il t9l E9+E9 RJ S '+(

"-r9[3;],')/0, t10l ,9 tEga-E9 R,I 9 | Z9<-t ,Xgl2i1

t11l +(O=ppXg.{-!r'X I tt MArg REPLACE! -7+,E9,t ' t)/M9
1727 -}0 r13l M9: tN1I D1ltEt V
$ v Z<-ia MATg B;C3D|E

tsIl E+pD<-7+(C,o) -0. C*(+/-C) + Ac+(-:\! '''=B) ^A=B+ 'B,A L21 z<.(r/^\r '=z)dz- 0 -r +(E,C)p(,QD".>tc<-l /D)\B v
v Z+REPLACES X;A:B:C ;D t1l z<-' ,,Q, ',t1l x,t0.5l | |
L2) ZIB<-(C^XeD)/rpX;f*(2 4 p' .r9 X9 t)t(d<-(^r\t t t t =x)^A<p D)/A<-(D<.tua')rX;l

t3l C<-(pZ)p 0 010
t4l clB, 12 47+7=1
t 5l z<-(,c) / ,z v

v R+A RJNS BtX

t1l X<-(-r+pA<(-2t 7 7 ,pA)pA)l-7+pB<.(-2+ I7 ,pB)pB
121 E<-(((1+p.),X)1e),t11((7rpB),X)"8 e
i Z<-A SPECJFI9 B iC iD;E t1l z+ o o ptl
12) .'(0=p8<-(8r t+t)+B)/0
t3l C*tr, ,6(-1+c\ I ')1c<-(+/^\r t=c)+c<,(+/^\BeALPHe)rB t4l 2..(-v / (((1'r0.4),8)'tA)^.=(E<-(7+pA)l pC)+c)l(I,pc)pc tsl z<-z RJ e(A RJNq C) SPECrlvg B v

APt g. g(. 9-alhqtbJ DOMAIN gg,// G"a7.

APL functions take arrays of numbers or characters as

arguments and. return arrays as correspond.ing results. More precisely, an APL function is a relationship between a

domain of permissible arguments and a range of corresponding results. Functions are classified according to their domains. For example: boolean functions have donains and a

range consisting solety of 0 and 1, arithmetic functions map nunbers to nunlf,ers, division excludes 0 as a right argument.

The domains of all scalar functions consist of numbers, and I'd like to suggest a sma1l set which is representative of the real number system.
First, choose a few interesting numbers:
s<-2,(*7) ,2.5,07
- an even nunber, e, a rational number, and pi; an integer, a nunber with a decimal part between zero and ,5, a decimal part of ,5, and a decimal part between .5 and 1.
There are as many nunbers between zero and 1 as there

are greater than 1. Extending s by v
s+(0+s),1,s
results in a set \trith a representative clistribution about 1. Real numbers are also alistributed slmmetricall-y about zeros s<-(0-s),o,s
The resulting 1g-element vector contains nunbers with
30 or so interesting properties, the bool-ean set, a subset of integers, and so on, antl is repreientative of the set of real nunlcers. It pxoves adequate for determining the domain and range of all API, functions.

COMPLEX E.8..//aO"mme// PRIMES gg-dt %.2V. \. Algorj-thms have appeared [1,2], v"ritten in A1gol, whj"ch

produce the complex primes within a given ringe. These algorithms are moderately complicated, each over thirty statements long. The present note gives a sequence of eight APL statements for producing a representation of a specifietl range of complex pr j-mes.
Fermat showed that a real prime of the form 1+4x,'y has a unique representation as the sum of two squares (see, for example, reference [3]). For example, 5=2 I+.*2 anil 13=2 3+,*2. This representation is related to the decomposition of a real prime of this kj-nd into comPlex prime factorcs. In other words, 5 and 13 are not prime in the complex tlomain. Thej-r decompositions (using an orthography proposed in [4]) ate s=2r7x2r-7 a\d, 13=2r3'<21-3. This decomposi-tion is unique to within associates (an associate of a number is a number differing fron the given number by a unit factort the complex unit.s are rt -1, oII, and ot-1). The complex primes are exhausted by these primes, the real Primes of the form 314x1y (which remain prine in the complex dornain) ' and the complex pr ime tlivisors of 2.

The complex prime divisors of 2 are 1f1 and its
associates, 7f-7, -7f7, and -1-f-1. Just as 2 and its associate -2 are unique among the real primes, j-n being the only even primes, so 111 anal its associates are unique amongt the complex primes in havj-ng real and imaginary parts with the same parity.

\jhhe construction given here provides a table ? j-ndicating

bf an entry of 1 at ?[F;1] that the complex integer with real part 2x,R and irnaginary par:L -7+2xI is prime ' and by an entry of 0 that it is composite. The table shows one of the primes; there are eight in all formed using the same ,q anal I: they are the number found j.n the table ancl its associates' and the conjugate of the nunber found in the tabLe and its associates. Since a complex prj-me (other than 1r1 and it;-Tssociates) always has real and j-maginary parts of opposite parity. and always has either an associate or an associate of its conjugate in the first quatlrant, we may choose as representat.ive of eight different complex primes the one having even, posltive real part, antl oddr positive inaginary part.
To construct a table rathich shows which are prime among
the complex integers with real Part less than or equal to M, and. imaginary part less than or equal to Iy, r"/e

- Proceed as
fol-lows. First, we need a list of the real prirnes up to

(M,ll)+.',2. Such a list may be obtained by using the functions p and. Qt expressed in the a@-notation [5,6]:
Pt2tL+u,Q 2
Q t Q a, (rtl 0xa", I V') / V1-K+ rK t L<K<-l / u t u

4 depends on Bertrandrs postulate (proven by chebyshev) [j] that a prime number wj-1l always be found between x and 2xx: for X greater than 3.
Next, proceed as folLows:

A<-(o =2l tM) / tM

B+(1=2l t)/rfi

E<-A*2
0<-B *2

S<-E o

, +0

I<-S eP (M,N)+.*2

the inforrnation in 7 may be displayed in a compact form by forming a companion table U, in which the o's antl 1rs in? are replaced. by spaces anal .rrs- From this, a table ,/ may be formed., with row and col]mn labels added to make readj.ng

easier:

U*t +t L7+T J

va-(t t ,1712 0r((p1) ,7)p2xtpA),(Q2 0v10
lo ((oB),7) o- 7+2x I oB),l7lLl

71-777
1357913579
2++++ +++
4+ + .++ +
6+ + + +
I ++r r +
10 ++ ++ + +r'
72.r++

such a table for a I of 6 rows and.

complex integers with even real Part irnaginary part from 1 to 19, is shown

10 colurnns (covering

from 2 to 12, and od.d in the figure;

to Proaluce
197 0

Harper E Bros. ,

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