"Slide and Divide" Explained

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I saw some pretty strange problem-solving heuristics arrivee at my desk while working as a tutor, but one of the worst was "slide and divide", a method of factoring quadratic polynomials with a non-unit leading coefficient. I never saw this method in the wild myself; my boss showed it to me after seeing it in a student's homework many years before.

Consider a quadratic polynomial with a non-unit leading coefficient such as $6x^2 + 13x + 5$
"Slide" the leading term to the right and multiply it to the constant term.

$$6x^2 + 13x + 5 \to x^2 + 13x + 30$$

This new quadratic polynomial factors more easily.

$$x^2 + 13x + 30 = (x + 3)(x + 10)$$

We now "divide" the leading coefficient out of the constant factors.

$$\left(x + \frac{3}{6}\right)\left(x + \frac{10}{6}\right)$$

Both of these fractions need to be simplified.

$$\left(x + \frac{1}{2}\right)\left(x + \frac{5}{3}\right)$$

The last step is to "clear" the fractions.

$$(2x + 1)(3x + 5)$$

This heuristic is full of operations that have no mathematical justification, from combining unlike terms to incorrectly dividing expressions with multiple terms. At the end of the process, however, we are indeed left with the correct factorization of the polynomial. Why does it work?
In truth, there is some interesting machinery hidden behind the "slide and divide" method. In this note we will recreate each step of the method in a mathematically-rigorous way.

Our first step is to factor out the leading term.

$$6x^2 + 13x + 5 = 6\left(x^2 + \left(\frac{1}{6}\right)13x + \left(\frac{1}{6}\right)5\right)$$

To help with factoring the inside expression, we separate another factor of $\frac{1}{6}$ out of the final term.

$$6x^2 + 13x + 5 = 6\left(x^2 + \left(\frac{1}{6}\right)13x + \left(\frac{1}{6}\right)^2 30\right)$$

The “slide and divide” method hides the leading coefficient as well as the fractions inside the expression.

We can now factor the inside fairly easily.

$$\begin{aligned} 6x^2 + 13x + 5 &= 6\left(x + \left(\frac{1}{6}\right)3\right)\left(x + \left(\frac{1}{6}\right)10\right) \\ &= 6\left(x + \frac{3}{6}\right)\left(x + \frac{10}{6}\right) \end{aligned}$$

This is where the “slide and divide” method introduces fractions, but the method presents these fractions as a division instead of coefficients used in factoring the polynomial. At this point the leading coefficient is still omitted from the method.

We can simplify the inner fractions as usual.

$$6x^2 + 13x + 5 = 6\left(x + \frac{1}{2}\right)\left(x + \frac{5}{3}\right)$$

When we multiply the leading coefficient back into the expression, the denominators of each fraction cancel.

$$6x^2 + 13x + 5 = (3x + 1)(2x + 5)$$

The “slide and divide” method never explicitly introduces the leading coefficient; it simply tells the student to multiply each factor by the denominator of the fraction it contains.

Though it produces a correct result each time, the “slide and divide” method does not present any deductive reasoning to the reader, and thus it provides no further insight into the nature of polynomial factorization. There is machinery behind it that works, but the method itself obfuscates that machinery. If anything, the method should be thought of as a kind of shorthand for the real factorization method, not a factorization method in its own right.

When learning to factor quadratic polynomials, students often struggle the most with expressions that have a non-unit leading coefficient, but there are much better methods to use than “slide and divide”. As it is taught, it’s simply a tool that students can use to give a correct answer on an exam, but it does not help those students understand why the answer is correct. It is a prime example of black box: a process that works but is opaque to the user.

At our tutoring facility, we did not allow students to use the method, even if it was included in their schools’ curriculum. If it is to be taught at all, I believe its nature as a shorthand method should be emphasized above all else.

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